Introduction to Scientific Computing
CME108/MATH114 – Summer 2020

Prerequisites

Single-variable calculus, especially Taylor expansions and the Intermediate Value Theorem. Exposure to linear algebra is highly recommended. Exposure to differential equations and partial derivatives will be helpful. Some programming experience will be assumed. While competence in a scientific computing language like MATLAB, Python, or Julia is not required, it would be put to good use.

Learning goals

How can I compute it? In this course we will investigate how to obtain numerical solutions for a wide gamut of mathematical problems encountered often in scientific and engineering contexts, like solving linear and non-linear equations, interpolating functions, approximating derivatives and integrals, locating function extrema, and simulating dynamical systems. We will not only develop and implement solution algorithms, but we will also prove theorems guaranteeing the performance of our methods.

Staff

Primary instructor (PI): Guillermo (Willie) Aboumrad. You can email me at any time at willieab@stanford.edu.

Teaching assistant (TA): Chris Lazarus. His email is clazarus@stanford.edu.

Format

We will loosely follow the "flipped classroom" model. Interactive(!) lecture modules will be available on our course website. We will offer two weekly hands-on interactive real-time sessions. We will work together through relevant examples, discuss applications or extensions of the theory covered in our lecture modules, and we will clarify doubts that may have come up as you studied the lecture modules. Both sessions in a given week will cover the same material, and only the second session will be recorded.

We will offer real-time sessions every Wednesday, from 9:30 to 10:30 AM (PDT) and every Thursday, from 4 to 5 PM (PDT).

Per university policy, we will use password-protected Zoom meetings for all real-time meetings (including office hours). Every meeting in a given week (including office hours) will be protected by the same password. We will change the meeting password each week. Your challenge is to complete week n's lecture modules to obtain week n's Zoom meetings password. Real-time sessions will be recorded and recordings will be available under the Cloud recordings tab on the Zoom page of our Canvas site.

Getting in touch

There are plenty of opportunities to contact the course staff. You are encouraged to join Willie's office hours to discuss problems and course material, and math more broadly.

We will offer office hours according to the following schedule. All times are listed in Pacific Time.

- Tuesday, 4 to 5 PM (Chris)
- Wednesday, 1 to 2 PM (Chris)
- Thursday, 5 to 6 PM (Willie)
- Friday, 10 to 11 AM (Willie)

We have also set up a Slack channel on Stanford's Slack network for your benefit. We encourage you to use it to discuss and collaborate with your peers. This is **not** a Piazza-like offering. The channel is meant to serve as a discussion board fostering communication and collaboration amongst peers. The course staff will **not** monitor the channel, but we **may** occasionally chime in.

We have set up course mailing lists. Please contact the course staff at `math114-sum1920-staff@lists.stanford.edu`. Please also look out for course announcements sent out to `math114-sum1920-students@lists.stanford.edu`.

Materials

The course textbook is Bradie's A Friendly Introduction to Numerical Analysis. It is available at the Stanford bookstore and on Amazon.com. (Library staff have informed me relevant chapters have been put on their high-priority scanning queue. I hope they are available online soon.)

Ascher and Greif's A First Course in Numerical Methods, available online through the Stanford library, is another great resource.

For a great introduction to numerical linear algebra, check out Bronson and Costa's Linear Algebra. This book will serve as a good supplement. For more advanced treatments, containing much material well beyond the scope of this course, Trefethen's Numerical Linear Algebra and Golub's Matrix Computations are phenomenal.

Assignments and Grading

We will assign weekly problem sets due before Sunday at 11:59 PM, with one exception. The first assignment is due before Monday, July 6th at 11:59 PM.

Homework assignments will make up 50% of your course grade. Recall that your graders can also make mistakes. If, upon careful review, you believe that you were graded erroneously, you have 72 hours after your grades are posted to Canvas to submit a regrade request via Gradescope.

There will be a short quiz on Monday, July 20 (Week 5), worth 10% of your course grade, to assess your progress after the first half of the course.

Lastly, a take-home final exam will make up the remaining 40% of your course grade. You will have approximately 72 hours to complete it, starting on Thursday, August 13 (last day of class).

Submissions will be handled via Gradescope and late work will generally **not** be accepted, except in highly extraordinary situations.

Bottom line:

- Weekly homework = 50%
- Quiz = 10%
- Final = 40%

Electronic Platforms

There are 3 sites to keep in mind.

Our course website contains our lecture modules, supplementary notes, and our homework assignments and their solutions. You may access the site at cme108.stanford.edu or math114.stanford.edu.

Canvas will keep track of grades and Zoom links for real-time meetings (including office hours).

Gradescope will manage submissions and regrade requests.

Topics

We will cover many interesting applications of the topics listed below. Some others will be developed as part of your homework assignments. The numbers next to a topic refer to relevant sections in Bradie [B] and in Ascher and Greif [AG].

1. Floating point number systems and floating point arithmetic ([B] 1.3, 1.4; [AG] 2.1, 2.2, 2.4)
2. Rootfinding
   (a) Bisection method ([B] 2.1, [AG] 3.2)
   (b) Fixed-point iteration schemes ([B] 2.3; [AG] 3.3)
   (c) Newton's method ([B] 2.4, 2.5; [AG] 3.4)
3. Linear systems (see [B] 3.0 or [AG] 4.1 for background)
   (a) Gaussian elimination ([B] 3.1, 3.2; [AG] 5.1)
   (b) LU decomposition ([B] 3.5, 3.6, 3.7; [AG] 5.2, 5.3, 5.4, 5.5)
   (c) Iterative methods ([B] 3.8, [AG] 7.2)
4. Non-linear systems

(a) Newton's method for systems ([AG] 9.1)

5. Interpolation

    (a) High-degree polynomial interpolation ([B] 5.1, 5.4; [AG] 10.1, 10.2, 10.3, 10.5, 10.6)
    (b) Cubic spline ([B] 5.5, 5.6; [AG] 11.1, 11.2, 11.3)

6. Numerical differentiation and quadrature rules

    (a) First and second order schemes ([B] 6.1, 6.2, 6.3; [AG] 14.1, 14.3, 14.5)
    (b) Newton-cotes schemes ([B] 6.4, 6.5; [AG] 15.1, 15.2)
    (c) Gaussian quadrature ([B] 6.6, [AG] 15.3)

7. Optimization

    (a) Gradient descent ([AG] 9.2.2)
    (b) Newton's method ([AG] 9.2.2)
    (c) Linear and Nonlinear Least-squares problems ([B] 3.10; [AG] 6.1, 9.2.4)

8. ODEs/PDEs

    (a) First order ODEs ([B] 7.1, [AG] 16.1)
    (b) Euler's methods ([B] 7.2, [AG] 16.2)
    (c) Finite difference schemes for boundary value problems ([B] 8.1, 8.2)
    (d) Finite elements and spectral decomposition (time permitting) ([AG] 16.8)